B.A. /B.Sc. Part-III (Honours) Examination, 2020 (1+1+1)
Subject: Mathematics
Paper: VII

Time: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any 4 questions from the following: 4×5=20

(a) Prove that $P(a < X \le b, c < Y \le d) = F(b,d) + F(a,c) - F(b,c) - F(a,d)$ where $F(x,y)$ denotes two-dimensional distribution function of (X,Y). 5

(b) The bivariate random variable (*X*,*Y*) follows the joint probability density function

$$f(x,y) = kx^2(8-y); x < y < 2x, 0 \le x \le 2$$
$$= 0; \text{ elsewhere}$$

Find (i) the value of k, (ii) the marginal probability density function of *X* & *Y* and (iii) the conditional probability density function $f_{x/y}(x/y)$. 1+2+2

(c) Obtain Bernoulli's theorem as a particular case of the law of large numbers for equal components. 5

(d) A radioactive source emits on the average 2.5 particles per second. Calculate the probability that 3 or more particles will be emitted in an interval of 4 seconds. 5

(e) Find the mean and variance of normal distribution. 2+3

(f) If $a_1(\neq 0), a_2(\neq 0), b_1, b_2$ are constants and $\rho(X,Y)$ be the corelation coefficient of *X* and *Y*, then prove that $\rho(a_1X + b_1, a_2Y + b_2) = \dfrac{a_1a_2}{|a_1||a_2|}\rho(X,Y)$. 5

2. Answer any 2 questions from the following: 2×5=10

(a) Fit a curve of the form $y = x^2 + ax + b$ to the following data by the method of least squares:

x	2	3	4	5	6
y	7.2	3.9	3.0	4.4	6.3

5

(b) If X and Y are independent variates, X being χ^2-distributed with *m* degrees of freedom and their sum $X+Y$ is χ^2- distributed with $(m+n)$ degrees of freedom, then show that Y is χ^2-distributed with *n* degrees of freedom. 5

(c) Find the maximum likelihood estimator of σ^2 for a normal (m,σ) population if *m* is known and show that the estimate is unbiased and consistent. 5

(d) The random variable X can take all non-negative integral values and

$$P(X = i) = p(1-p)^i, i = 0,1,2.....$$

where $p(0 < p < 1)$ is a parameter. Find the maximum likelihood estimator of p on the basis of a sample of size n from the population of X. 5

3. Answer any 4 questions from the following: 4×5=20

(a) Show that the set of all convex combinations of a finite number of points is a convex set. 5

(b) Solve the following LPP graphically: 5

$$\begin{aligned} \text{Maximize} \quad & z = 3x_1 + 2x_2 \\ \text{subject to} \quad & x_1 + x_2 \le 2 \\ & x_1 + 4x_2 \le 4 \\ & 2x_1 + 3x_2 \le 6 \\ \text{and} \quad & x_1, x_2 \ge 0 \end{aligned}$$

(c) Formulate the dual of the following LPP: 5

$$\begin{aligned} \text{Maximize} \quad & z = 2x_1 + 3x_2 + 4x_3 \\ \text{subject to} \quad & x_1 - 5x_2 + 3x_3 = 7 \\ & 2x_1 - 5x_2 \le 3 \\ & 3x_1 - x_3 \ge 5 \\ & x_1, x_2 \ge 0 \text{ and } x_3 \text{ is unrestricted in sign.} \end{aligned}$$

(d) Solve, if possible, by dual simplex method, the following LPP: 5

$$\begin{aligned} \text{Minimize} \quad & z = 6x_1 + 11x_2 \\ \text{subject to} \quad & x_1 + x_2 \ge 11 \\ & 2x_1 + 5x_2 \ge 40 \text{ and } x_1, x_2 \ge 0 \end{aligned}$$

(e) Find an initial basic feasible solution of the following transportation problem by Vogel's approximation method. 5

	D_1	D_2	D_3	D_4	Supply
O_1	11	13	17	14	250
O_2	16	18	14	10	300
O_3	21	24	13	10	400
Demand	200	225	275	250	950

(f) Solve the following game graphically: 5

Player A

Player B	2	2	3	-1
	4	3	2	6